WEEKLY TEST RANKER'S BATCH TEST - 10 RAJPUR
SOLUTION Date 01-12-2019

[PHYSICS]

1. We are given that a particle of mass m is located in a one dimensional potential field and the potential energy is given by $V(x) = A(1 - \cos px)$.

So, we can find the force experiened by the particle as

$$F = -\frac{dV}{dx} = -Ap\sin px$$

For small oscillations, we have

$$F \approx -Ap^2x$$

Hence, the acceleration would be given by

$$a = \frac{F}{m} = -\frac{Ap^2}{m}x$$

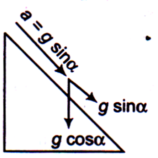
Also we know that

$$a = \frac{F}{m} = -\omega^2 x$$

So, $$\omega = \sqrt{\frac{Ap^2}{m}}$$

or $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{Ap^2}}$$

2. We are given that the simple pendulum of length l is hanging from the roof of a vehicle which is moving down the frictionless inclined plane.



So, it's acceleration is $g\sin\theta$. since vehicle is accelerating a pseudo force $m(g\sin\theta)$ will act on bob of pendulum which cancel the $\sin\theta$ component of weight of the bob.
Hence we can say that the effective acceleration would be equal to

$$g_{\text{eff}} = g\cos\alpha$$

Now the time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$$

$$= 2\pi\sqrt{\frac{l}{g\cos\alpha}}$$

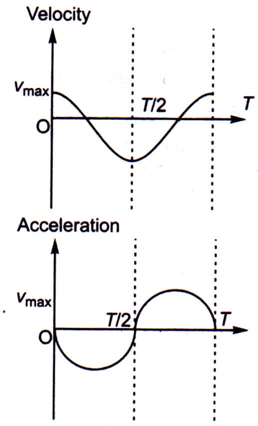
3. The radius of particle is 3. Which is maximum, so the amplitude of simple Harmonic motion is 3 cm.

4. Phase difference $\Delta\phi = \phi_1 - \phi_2$

$$= \frac{3\pi}{6} - \frac{\pi}{6}$$

$$= \frac{2\pi}{6} = \frac{\pi}{3}$$

5. In SHM, the acceleration is ahead of velocity by a phase angle $\frac{\pi}{2}$.



$$v = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$a = A\omega^2\sin(\omega t + \pi)$$

6. The total energy a particle executing SHM

$$= \frac{1}{2}m\omega^2A^2$$

The PE of the particle at a distance x from the equilibrium position

$$= \frac{1}{2}m\omega^2x^2$$

From the question, $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}\left(\frac{1}{2}m\omega^2A^2\right)$

$\Rightarrow$ $$x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$$

7. $v_{max} = \alpha\omega$ and

Maximum acceleration $= \omega^2 a = \left(\frac{v}{a}\right)^2 a = \frac{v^2}{a}$

8. The average acceleration of a particle performing SHM over one complete oscillation is zero.

9. Total energy in SHM

$$E = \frac{1}{2} m\omega^2 a^2, \text{ (where, } a = \text{amplitude)}$$

Kinetic energy $K = \frac{1}{2} m\omega^2 (a^2 - y^2)$

$$= E - \frac{1}{2} m\omega^2 y^2$$

when $y = \frac{a}{2}$

$$\Rightarrow \quad K = E - \frac{1}{2} m\omega^2 \left(\frac{a^2}{4}\right) = E - \frac{E}{4}$$

$$E = \frac{3E}{4}$$

10. Let x be the point where KE = PE

Hence $\frac{1}{2} m\omega^2 (a^2 - x^2) = \frac{1}{2} m\omega^2 x^2$

$$2x^2 = a^2, \; x = \frac{a}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ cm}$$

11. For a body executing SHM, velocity,

$$v = \sqrt{\omega^2 (a^2 - y^2)}$$

we have $10^2 = \omega^2 (a^2 - 4^2)$

and $8^2 = \omega^2 (a^2 - 5^2)$

So, $10^2 - 8^2 = \omega^2 (5^2 - 4^2) = (3\omega)^2$

or $6 = 3\omega$

or $\omega = 2$

$\because$ Time, $t = \frac{2\pi}{\omega}$

$\therefore$ $t = \frac{2\pi}{2} = \pi$ second

12. $\frac{d^2 x}{dt^2} + 16x = 0$

$\therefore$ $\omega^2 = 16 \Rightarrow \omega = 4$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

13. The stretched spring oscillates in accordance with the Hooke's law which goes as

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$$

$\Rightarrow$ $\omega^2 = \frac{k}{m}$

$\Rightarrow$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$= \frac{1}{2} 2\pi \sqrt{\frac{8\pi^2}{0.5}}$$

$$= 2 \text{ Hz}$$

14. The time period of the iron ball would be given by,

$$P = 2\pi\sqrt{\frac{M}{K}}$$

So, if mass of the ball is increased to 4 times of its initial mass, then the new period becomes

$$P' = 2\pi\sqrt{\frac{4M}{K}} = 2P$$

15. We have

$$x = -0.3 \sin\left(t + \frac{\pi}{4}\right)$$

Comparing with the general equation

$$x = x_0 \sin(\omega t + \phi)$$

So, $x_0 = 0.3$, $\omega = 1$, $\phi = \frac{\pi}{4}$

Hence, $2\pi f = 1$

$\Rightarrow$ $f = \frac{1}{2\pi}$

16.

17. Gravity, $g = \frac{GM}{R^2}$ (G is **constant**)

$\therefore$ $\frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2}$

$\Rightarrow$ $\frac{g_e}{g_p} = \frac{2}{1}$

Also, $T \propto \frac{1}{\sqrt{g}}$

$\Rightarrow$ $\frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}}$

$\Rightarrow$ $\frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$$T_p = 2\sqrt{2} \text{ s}$$

18. Displacement-time equation of the particle will be,

$$x = A\cos\omega t$$

Given that; $x_1 = A\cos\omega$

$x_2 = A\cos 2\omega$

and $x_3 = A\cos 3\omega$

Now, $\dfrac{x_1 + x_3}{2x_2} = \dfrac{A(\cos\omega + \cos 3\omega)}{2A\cos 2\omega}$

$= \dfrac{2A\cos 2\omega\cos\omega}{2A\cos 2\omega} = \cos\omega$.

$\therefore$ $\omega = \cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right) = \dfrac{2\pi}{T}$

or $T = \dfrac{2\pi}{\omega}$, where $\omega = \cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)$.

19. A
20. A
21. A
22. D

23. For simple harmonic motion, $v = \omega\sqrt{a^2 - x^2}$

When $x = \dfrac{a}{2}, v = \omega\sqrt{a^2 - \dfrac{a^2}{4}} = \omega\sqrt{\dfrac{3}{4}a^2}$

As $\omega = \dfrac{2\pi}{T}$,

$\therefore$ $v = \dfrac{2\pi}{T}.\dfrac{\sqrt{3}}{2}a$

or $v = \dfrac{\pi\sqrt{3}a}{T}$

24. For a particle to execute simple harmonic motion, its displacement at any time t is given by:

$$x(t) = a(\cos\omega t + \phi)$$

Where, a = amplitude; ω = angular frequency; ϕ = phase constant

Let us choose $\phi = 0$

$\therefore$ $x(t) = a\cos\omega t$

Velocity of a particle, $v = \dfrac{dx}{dt} = -a\omega\sin\omega t$

Kinetic energy of a particle is, $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}ma^2\omega^2\sin^2\omega t$

Average kinetic energy $\langle K\rangle = \langle \dfrac{1}{2}ma^2\omega^2\sin^2\omega t\rangle$

$= \dfrac{1}{2}m\omega^2a^2\langle\sin^2\omega t\rangle$

$= \dfrac{1}{2}m\omega^2a^2\left(\dfrac{1}{2}\right)\left[\because \langle\sin^2\theta\rangle = \dfrac{1}{2}\right]$

$= \dfrac{1}{4}ma^2(2\pi\nu)^2$ $[\because \omega = 2\pi\nu]$

$= \pi^2ma^2\nu^2$.

25. A
26. B
27. C
28. A

29.

$y = 4\cos^2(t/2)\sin(1000t)$

$= 2[2\cos^2(t/2)\sin(1000t)]$

$= 2(1 + \cos t)\sin(1000t)$

$= 2\sin(1000t) + 2\sin(1000t)\cos t$

$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$

i.e., the given wave represents the superposition of three waves.

30. Resultant amplitude,

$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{A^2 + A^2 + 2A^2\cos\phi}$

But $A_R = A$

$\therefore$ $A = \sqrt{2A^2(1 + \cos\phi)}$

$= \sqrt{4A^2\cos^2\dfrac{\phi}{2}} = 2A\cos\dfrac{\phi}{2}$

or $\cos\dfrac{\phi}{2} = \dfrac{1}{2}$ or $\phi = \dfrac{2\pi}{3}$.

31. C

32. (b) Let the line joining AB represents axis 'r'.By the conditions given 'r' coordinate of the particle at time t is

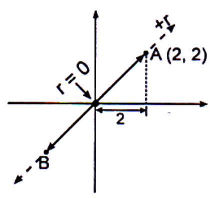
$r = 2\sqrt{2}\cos\omega t$

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2}\pi$

$\therefore$ $r = 2\sqrt{2}\cos\pi t$

$x = r\cos 45^o = \dfrac{r}{\sqrt{2}} = 2\cos\pi t$

$\therefore$ $a_x = -\omega^2x = -\pi^2 2\cos\pi t$ $\therefore$ $F_x = ma_x = -4\pi^2\cos\pi t$



33. (c) Both the spring are in series

$\therefore$ $K_{eq} = \dfrac{K(2K)}{K + 2K} = \dfrac{2K}{3}$

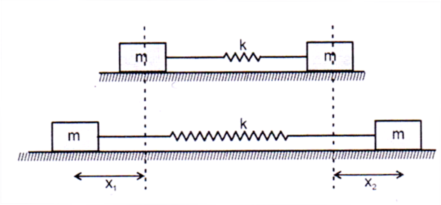
Time period

$T = 2\pi\sqrt{\dfrac{\mu}{K_{eq}}}$ where $\mu = \dfrac{m_1m_2}{m_1 + m_2}$

Here $\mu = \dfrac{m}{2}$ $\therefore$ $T = 2\pi\sqrt{\dfrac{m}{2}.\dfrac{3}{2K}}$

$= 2\pi\sqrt{\dfrac{3m}{4K}}$

Method II



$\therefore \quad mx_1 = mx_2 \Rightarrow x_1 = x_2$

force equation for first block;

$$\frac{2k}{3}(x_1 + x_2) = -m\frac{d^2x_1}{dt^2}$$

Put $x_1 = x_2 \Rightarrow \; \Rightarrow \frac{d^2x_1}{dt^2} + \frac{4k}{3m} \times x_1 = 0 \qquad \Rightarrow \; \omega^2 = \frac{4k}{3m}$

$$\therefore \quad T = 2\pi\sqrt{\frac{3m}{4k}}$$

34. (b) $\frac{I_1}{I_2} = \frac{a_1^2 f_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$

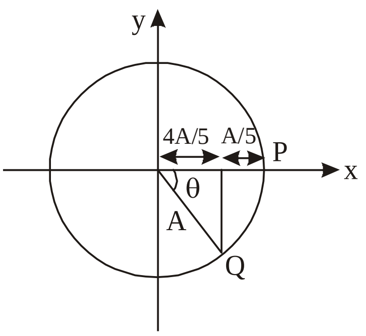
41. (b) Particle is strating from rest i.e., from one of its extreme position.

As particles moves a distance $A/5$, we can represent it on a circle as shown.

$$\cos\theta = \frac{4A/5}{A} = \frac{4}{5}$$

$$\theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\omega t = \cos^{-1}\left(\frac{4}{5}\right)$$



$$t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

Alternatively

As starts from rest i.e., from extreme position $x = A\sin(\omega t + \phi)$

At $t = 0$; $x = A$

$$\Rightarrow \; \phi = \frac{\pi}{2}$$

$$\therefore \quad A - \frac{A}{5} = A\cos\omega t$$

$$\frac{4}{5} = \cos\omega t$$

$$\Rightarrow \; \omega t = \cos^{-1}\frac{4}{5};$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

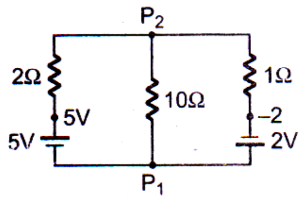
36. **Charge** $Q = \int_0^2 (3t^2 + 2t + 5)\,dt$

$= [t^3 + t^2 + 5t]_0^2$

$= 8 + 4 + 10 = 22$ C

37. Let potential of P_1 be 0 V and potential of P_2 be V_0.

Now, apply KCL at P_2.



$$\frac{V_0 - 5}{2} + \frac{V_0 - 0}{10} + \frac{V_0 - (-2)}{1} = 0$$

or $\quad V_0 = \frac{5}{16}$

38. From the given circuit,

$V_A - (6 \times 2) - 12 - (9 \times 2) + 4 - (5 \times 2) = V_B$

or $\quad V_A - 12 - 12 - 18 + 4 - 10 = V_B$

or $\quad V_A - V_B = 48$ volt

39. As the PD between 4 Ω and 3 Ω (in parallel) are the same,

4×1 amp $= 3 \times i_1$ or $i_1 = \frac{4}{3}$ A (Let i_1 = Current in 3 Ω resistance)

Total resistance of 4 Ω and 3 Ω = 12/7 Ω

Current in MQP (upper branch) $= 1 + \frac{4}{3} = \frac{7}{3}$ A

$\therefore$ PD (across upper branch) $= \frac{12}{7} \times \frac{7}{3} = 4$ V

Current in $MNP = \frac{4}{1.25} = \frac{4 \times 4}{5} = \frac{16}{5}$ A

$\therefore$ PD across 1 Ω $= \frac{16}{5}$ A $\times 1\,\Omega$

$= \frac{16}{5}$ volt $= 3.2$ volt

40. Terminal voltage, $V = E - Ir$

$V = 10 - 0.5 \times 3 = 10 - 1.5 = 8.5$ V

41. Applying Kirchhoff's loop rule to give *mgh*, we get;

$$-3i - 10i - 3i - 5.2 + 10 = 0$$

or $i = 0.3$ amp

42. Since, given circuit is in the form of Wheatstone bridge,

$$\frac{1}{R_{eq.}} = \frac{1}{(4+2)} + \frac{1}{(6+3)}$$

$\therefore \quad R_{eq.} = \frac{18}{5}$

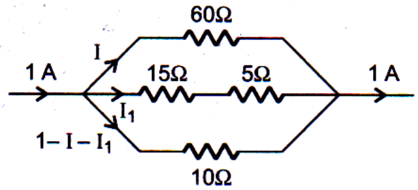
$\therefore \quad V = i\,R_{eq.} \quad$ or $\quad i = \frac{V}{R_{eq.}} = \frac{5V}{18}$

43. In the given circuit three resistances R_2, R_4 and R_3 are in parallel, hence

$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_3}$$

$$= \frac{1}{50} + \frac{1}{50} + \frac{1}{75} = \frac{75 + 75 + 50}{50 \times 75}$$

44. Using voltage is same in all three branches:



45. The given network of resistances between points *A* and *B* is equivalent to a balanced Wheatstone bridge.

Hence, $\quad R_{AB} = R$

and current flowing in *AFCEB* $= \frac{V}{2R}$

[CHEMISTRY]

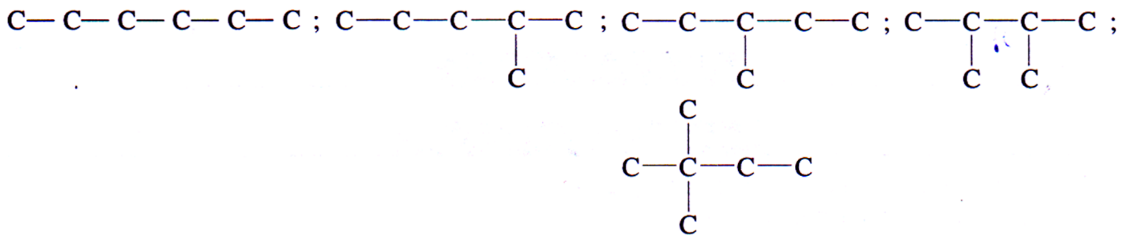
46. The structure of given compounds are

2-Butene	$CH_3—CH{=}CH—CH_3$	2-Butanol	$CH_3CH_2CH(OH)CH_3$
2-Butyne	$CH_3—C{\equiv}C—CH_3$	Butanal	$CH_3CH_2CH_2CHO$

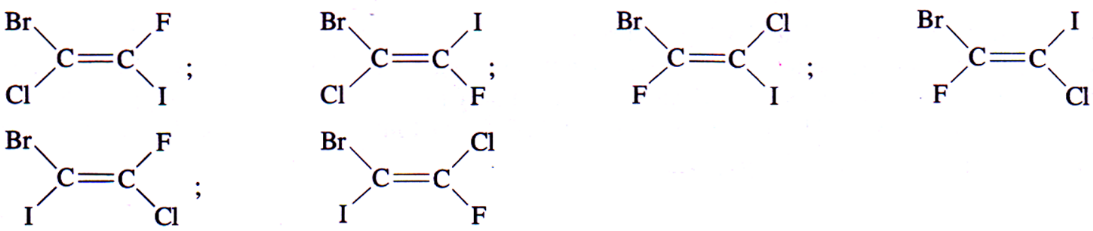
Only 2-Butene shows *cis-trans* isomerism.

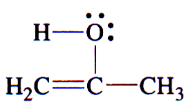
47. Different formulae implies different molar masses.

48. The skeletons of carbon in C_6H_{14} are as follows



49. The six compounds will be



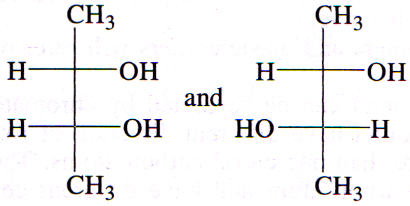
50. The enolic form of acetone is  . It contains 9 sigma bonds, 1 pi bond and two lone pairs of electron on oxygen.

51. Isomers which can be interconverted through rotation around a single bond are known as conformers.

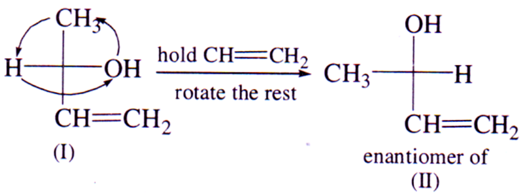
52 Only one optically active compound $H_3CCH(Br)CHBr_2$ is possible.

53. The compounds I and II are enantiomers. Rotation of compound II by 180° produces mirror image of compound I.

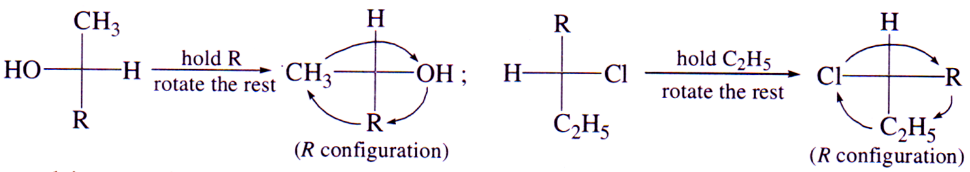
54. For the compound $CH_3CH(OH)CH(OH)CH_3$, two optically active seteroisomers are possible.



55. The structures (I) and (II) represent enantiomers.



56. We have



The compound is named as $(2R_1 3R)$-3-chloro-2-pentanol.

57. The number of stereoisomers will be 16 (= 2^4).

58. The two enantiomers differ in their optical activities.